CONVECTIVE DIFFUSION IN POROUS MEDIA

(KONVEKTIVNAIA DIFFUZIIA V PORISTYKH SREDAKH)

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The motion of a fluid through a plane porous medium is accompanied by an intensive mixing process, which it is possible to examine by introducing into the stream an admixture of particles which do not change the properties of the fluid or the parameters of the motion, do not adhere to the walls of the pores, and which move with the ambient velocity at each point of the porous region.

The hollow intersecting pores may be regarded as a system of porous channels, each of which communicates with all its neighbours. An example of such a medium is provided by a layer of sand.

The local velocity of motion of the particles of fluid is a random function of position in the porous region [1]. The propagation of the indicator which has been added to the stream is therefore governed by a law which is to a certain extent analogous to the law of diffusion of an inert ingredient in a turbulent stream.

The construction of a capillary model of the mechanism of such diffusion, which can be called convective diffusion, was presented in an earlier paper [2]. It still has some illustrative value because of the inadequacy of our knowledge of the relationships connecting the hydraulic characteristics of porous channels.

In the present paper we introduce certain averaged parameters of an isotropic homogeneous porous medium which determine the character of convective diffusion.

1. Let us consider a stream of homogeneous fluid in an isotropic homogeneous porous medium. Isotropy is assumed in the sense of invariance of the distribution of the statistical characteristics the medium relative to bulk rotations or mirror reflections. The flow of the fluid will be defined by the mean velocity vector. The average of the velocity, as well as that of the characteristics of the porous medium, is taken over a representative group of samples. Moreover, it is assumed that the results of the averaging do not depend upon the choice of points in the porous region, i.e. it is possible to take a point, located on any surface intersecting the medium, on any arbitrary curve described in the medium. Thus, for example, we can average over all points of the porous region located on an arbitrary plane section of an elementary macrovolume. It is assumed that we can choose an elementary macrovolume such that it will contain a representative group of samples, whilst the averaged characteristics of the current are uniform throughout its extent.

A fluid particle at different instants of time will move with changing velocity, specifically as a result of the intercommunication between the hollow pores since, if the porous region were constructed of continuous tubes which did not communicate with one another, then the particle would not pass through the whole representative group of samples (see Section 4).

If at the initial instant of time the particle was located at the origin of a system of coordinates moving with the mean velocity, then after time t its coordinates will be

$$x_\alpha(t) = \int_0^t v_\alpha(\tau)\,d\tau = \int_0^t [u_\alpha(\tau) - \bar{u}_\alpha]\,d\tau \tag{1.1}$$

where $u_\alpha(\tau)$ is the component of velocity along the axis α at the moment of time τ; there and henceforth the bar denotes the averaged value of the quantity.

The expression (1.1) is the integral of the random function $v_\alpha(t)$. From the limit theorem in the theory of probability it follows that this random integral has a distribution close to the normal distribution for an arbitrary distribution law of the function $v_\alpha(t)$, if $v_\alpha(t)$ assumes statistically independent values when separated by certain sufficiently small intervals of time $\tau_0 (t \gg \tau_0)$.

If at an arbitrary instant of time there exists a three-dimensional normal distribution, i.e. the probability density of finding the particle at the point with coordinates x_1, x_2 and x_3 has the form

$$\psi(x_1, x_2, x_3, t) = \frac{(2\pi)^{-3/2}}{\sqrt{\overline{x_1^2}\,\overline{x_2^2}\,\overline{x_3^2}}} \exp\left\{-\frac{1}{2}\sum_\alpha \frac{x_\alpha^2}{\overline{x_\alpha^2}}\right\} \tag{1.2}$$

then, regarding ψ as the relative concentration of the added particles, we see that (1.2) is the solution of the diffusion equation for a type of instantaneous source:

$$\frac{\partial\psi}{\partial t} = \sum_\alpha \frac{\partial}{\partial x_\alpha}\left(D_\alpha \frac{\partial\psi}{\partial x_\alpha}\right) \tag{1.3}$$

where D_α is the axial component of the diffusion coefficient:

$$D_\alpha = \overline{x_\alpha^2} / (2t) \qquad (1.4)$$

It appears that such a statistical approach to the consideration of the motion of particles in a porous medium was first made by Scheidegger [3], who assumed that the medium consisted of perfectly identical pores and so reduced the problem to the model problem of randomly wandering particles.

The coefficient of diffusion along the axes is different, therefore it must be a tensor of the second rank. In equation (1.3) the principal axes of this tensor are taken as the system of coordinates.

2. If in an isotropic homogeneous porous medium there is a homogeneous stream of fluid (the direction of the stream being constant), then the coefficient of diffusion, characterising the mixing process described in the foregoing section, is invariant relative to a rotation about the direction of the mean velocity, and to mirror reflections relative to planes including the mean velocity vector or perpendicular to this vector. In this case [4] the coefficient of diffusion will have the form of an axisymmetric isotropic point tensor

$$D_{ij} = A\bar{u}_i^\circ \bar{u}_j^\circ + BI_{ij} \qquad (2.1)$$

where A and B are constants, u_i^0 is the component of the perturbation of mean velocity and I_{ij} is the unit tensor.

In the case of turbulent diffusion in the field of a homogeneous stream the coefficient of diffusion is an isotropic tensor $D_{ij} = BI_{ij}$, since the entire region of mixing moves as a whole with the mean velocity; in the region of mixing, the direction of the vector of mean velocity is on a par with other directions. In the case now being considered the mixing region – the porous medium – is essentially fixed, whilst the current through it has an axis of symmetry – the direction of mean velocity.

Let us consider the most important motion – the motion governed by D'Arcy's law, in which inertial forces can be neglected [5]. From the characteristics of such motions, the mean velocity u, fluid viscosity μ and a certain characteristic length of the porous medium, it is impossible to construct any dimensionless combination, and evidently the coefficient of diffusion, having the dimension $cm^2 sec^{-1}$, is equal [6], up to a dimensionless constant multiplier, to the product of the mean velocity and the characteristic length already mentioned.

In so far as the diffusion process under consideration is not one-dimensional, the characteristic length will, generally speaking, not be a scalar quantity, but must be a certain tensor. We shall call this the dispersion tensor of the porous medium. Since the medium is isotropic, then the point dispersion tensor must be a tensor of even rank [4]. The diffusion coefficient is then represented in the form of the product of dispersion tensor with a quantity connected with the mean velocity vector and having the dimension of a velocity.

If we take the diffusion coefficient in the form of the product of a certain isotropic tensor of the second rank with the modulus of the vector of mean velocity, then all the components of the diffusion coefficient are identical: the diffusion coefficient is a spherical tensor. The mixing is isotropic, and the constant A of expression (2.1) must be equal to zero. This type of representation is adopted in relation to turbulent diffusion in the field of a homogeneous stream [7].

We can represent the diffusion coefficient in the form of the product of a scalar, representing dispersion in the porous medium, with a tensor consisting of the components $\bar{u}_k$ and $\bar{u}_l^0$. It is easy to show that then the dispersion will arise only in the direction of the vector of mean velocity whilst, in the directions orthogonal to this, it will be zero: in the expression (2.1) we must have $B = 0$. This representation is adopted in the capillary models of the diffusion process in porous media [2].

The dispersion tensor of a porous medium can be given in the form of a tensor of the fourth rank, which by virtue of the isotropy of the medium [4] is characterised by three constants:

$$Q_{ijkl} = H_1 I_{ij} I_{kl} + H_2 I_{ik} I_{jl} + H_3 I_{il} I_{jk}, \qquad [Q_{ijkl}] = [H_i] = \text{см} \tag{2.2}$$

Then the components of the diffusion coefficient are determined in the following way:

$$D_{ij} = Q_{ijkl} \bar{u}_k \bar{u}_l^\circ = Q_{ijkl} \bar{u}_k \bar{u}_l / |\bar{u}| \tag{2.3}$$

where summation is to be carried out over the repeated indices.

Since the diffusion coefficient is a symmetric tensor, then $H_2 = H_3$ by virtue of the equivalence of the indices k and l. It is easy to show that the expression (2.3) is equivalent to formula (2.1), with

$$B = H_1 |\bar{u}|, \quad A = 2H_2 |u|$$

If we take for the dispersion tensor a tensor of higher even rank, then the coefficient of diffusion is equal to

$$D_{ij} = Q_{ijkl\ldots m} \bar{u}_k \bar{u}_l^\circ \ldots \bar{u}_m^\circ \tag{2.4}$$

It can be shown that (2.4) in the general case reduces to the expression (2.1), and by virtue of the isotropy of the medium and the equivalence of the indices, over which the summation occurs, it is determined by two constants.

Accordingly, formula (2.1) for the diffusion coefficient is very general. The physical significance of the two postulated constants A and B or H_1 and H_2 is to be explained on the basis of the representation made as to the mechanism of mixing in porous media.

3. At each point of the porous region the vector of mean velocity is modified in a random manner into the local velocity. Moreover, generally speaking, the direction of the velocity as well as the value of its modulus will be changed. The transformation has the form:

$$u_i = T_{ij}\bar{u}_j \tag{3.1}$$

The components of the tensor T_{ij} are random quantities having different values for each sample (at an arbitrary point of the porous region). We shall call the tensor T_{ij} the local tensor of the porous medium. We shall consider below those flows for which inertial forces can be completely neglected, whence it follows from dimensional analysis that the components T_{ij} do not depend upon the magnitude of the mean velocity.

Moreover, we make the following hypothesis: the components of the local tensor of the porous medium do not depend upon the direction of the mean velocity.

From averaging relation (1.1) it is obvious that $\bar{T}_{ij}$ is the unit tensor. The dispersion velocity (when projected on the axes a) is expressed by the product of components of the fourth rank, characterising the porous medium:

$$D(u_\alpha) = \overline{(u_\alpha - \bar{u}_\alpha)^2} = (\overline{T_{\alpha i}T_{\alpha j}} - \bar{T}_{\alpha i}\bar{T}_{\alpha j})\,\bar{u}_i\bar{u}_j = (\overline{T_{\alpha i}T_{\alpha j}} - I_{\alpha i}I_{\alpha j})\,\bar{u}_i\bar{u}_j \tag{3.2}$$

By virtue of the isotropy of the porous medium and the equivalence of the indices i and j all the averaged products $T_{ai}T_{aj}$ will have the form:

$$\overline{T_{\alpha i}T_{\alpha j}} = C_1 I_{\alpha\alpha} I_{ij} + 2C_2 I_{\alpha i} I_{\alpha j} \tag{3.3}$$

i.e. all the products in which $i \neq j$ will vanish. The expression (3.2) can be simplified: the factor in the parentheses is the dispersion of the components T_{ai}

$$D(u_\alpha) = (\overline{T_{\alpha i}T_{\alpha i}} - I_{\alpha i})\,\bar{u}_i^2 = D(T_{\alpha i})\,\bar{u}_i^2 \tag{3.4}$$

Now, making use of the relation (1.1), let us find the mean square of the transport, or dispersion, in a moving system of coordinates:

$$\overline{x_\alpha^2} = \int_0^t\int_0^t \overline{v_\alpha(\tau_1)\,v_\alpha(\tau_2)}\,d\tau_1\,d\tau_2 \tag{3.5}$$

For long duration the dispersion (3.5) can be represented in the following form [7]:

$$\overline{x_\alpha}^2 = 2D(u_\alpha)L_\alpha t \tag{3.6}$$

where L_α is a quantity analogous to the Lagrange scale of turbulence.

Starting from dimensional considerations, let us set

$$L_\alpha = L = l/|\bar{u}| \tag{3.7}$$

where l is a certain "mixing length" of the porous medium – a scalar by virtue of its isotropy.

Making use of the relation (1.4) and taking account of the fact that the coefficient of diffusion is referred to its principal axes, we obtain

$$D_{11} = D_1 = D(u_1)l/|\bar{u}|, \quad D_{22} = D_2 = D(u_2)l/|\bar{u}|$$
$$D_{33} = D_3 = D(u_3)l/|\bar{u}| \tag{3.8}$$

i.e. the properties of the porous medium which are under consideration are characterised by a tensor of the fourth rank, and the relations (3.8) and (2.4) must be equivalent. The following relation therefore holds:

$$H_1 = C_1 l, \qquad H_2 = 0{,}5(2C_2 - 1)l \tag{3.9}$$

Moreover, (3.10)

$$B = H_1|\bar{u}| = D(T_{\alpha i})l|\bar{u}|, \quad A = 2H_2|\bar{u}| = [D(T_{\alpha\alpha}) - D(T_{\alpha i})]l|\bar{u}|, \quad \alpha \neq i$$

Accordingly, the coefficient of diffusion, when referred to the principal system of coordinates, has the form:

$$D_1 = D(T_{11})l\bar{u} = \lambda_1\bar{u}, \quad \bar{u} = \bar{u}_1, \quad \bar{u}_2 = \bar{u}_3 = 0 \tag{3.11}$$
$$D_2 = D(T_{12})l\bar{u} = \lambda_2\bar{u}, \qquad D_3 = D_2$$
$$D_{ij} = 0 \text{ when } i \neq j$$

4. For flows subject to D'Arcy's law the quantities T_{ij}, H and l are determined only by the structure of the porous medium. Similar results are obtained for flows subject to the quadratic law [8], when the inertial forces only are allowed for, although the numerical values of the quantitites are, generally speaking, different in the two cases. In the case where there are both viscous and inertial forces, these quantities will depend upon the Reynolds number, i.e. upon the velocity. This is confirmed by experiments – the coefficient of diffusion proves to be proportional to the mean velocity of the flow in the region of validity of D'Arcy's law [9], and in the region of validity of the quadratic law of filtration (with the Reynolds number greater than 500) [10], whilst in the intermediate zone (Reynolds number from 100 to 500) the dependence of the coefficient of diffusion upon velocity becomes more complex [10]. In

the paper referred to [10], the characteristic linear dimension was taken to be the diameter of the added particles.

If there arises the necessity of taking account of molecular diffusion, then on the assumption of no correlation between it and convective diffusion the effective diffusion coefficient is

$$D_\alpha^* = D_\alpha + D_0 \tag{4.1}$$

The coefficient of molecular diffusion D_0 usually turns out to be many times smaller than D_α.

The supposition of no correlation between these two processes is justified for porous media with intercommunicating pore spaces, for which the results of Section 1 are also verified.

In a number of papers concerning diffusion in a filtrating fluid, for example [11], deductions are made concerning the proportionality of the coefficient of diffusion to the square of the velocity of filtration. In these studies the porous medium is represented as a system of capillaries which are isolated from one another, penetrating continuously from one face of the medium to the other. Taylor [12] showed that the consequence of a steady parabolic distribution of velocity across the capillary section together with lateral molecular diffusion was that the diffusion coefficient for laminar flow in the capillary had the form:

$$D^* = (u_0^2 d^2) / (\gamma D_0) \tag{4.2}$$

where D_0 is the coefficient of molecular diffusion in the fluid, d is the diameter of the capillary, u_0 is the maximum velocity across the section and γ is a certain numerical parameter. If this result is applied to each capillary of the system, then the result referred to above is indeed obtained. Here we have complete correlation between the molecular and convective diffusion.

In porous media with intercommunicating pore spaces, however, the fluid particles which are moving in one of the pores along its axis, for example, and therefore with the maximum possible velocity, will generally speaking not have the maximum velocity in the next pore but will take up some other velocity.

We notice that from certain intuitive arguments it follows that the quantity l must be about half the diameter of the grains for uncemented porous media. For cemented porous media it is about the mean value of the segments of an arbitrarily constructed straight line, included between the points of intersection of this line with the section contours of the framework of the porous medium. If the porous channels in the medium could be separated, then l would be equal to half the length of the average

[2]. For a more precise definition of l, however, and thence also of the dispersion velocity involved in the coefficient of diffusion, it is necessary to introduce supplementary theoretical or experimental evidence.

It can be assumed that two porous media have similar porous voids if they are compounded in an identical way from particles of a given shape but different sizes; for simplicity we shall assume the size of the particles to be uniform in each medium, although the same argument could be extended also to the case with any practical particle composition. These media will differ only in the linear scale, and consequently the flow of the fluids in them is completely similar in the case of identity of the Reynolds numbers. Therefore the ratio of the viscosities of the fluids must be equal to the reciprocal of the ratio of the diameters of the particles (δ' and δ''). Then we can say that the dispersion components T_{ij} in such media are equal.

Hence, for equal mean flow velocities we must have the relation

$$\frac{D_\alpha'}{D_\alpha''} = \frac{l'}{l''} = \frac{\delta'}{\delta''} \qquad (4.3)$$

where primes are used to distinguish between quantities related to the one medium or the other.

The experimental testing of equation (4.3) is completely possible, for example on spheres of different diameters, and its accomplishment either gives confirmation of the arguments developed here or exposes the need for corrections. Accordingly, such an experiment can confirm the validity of the relationship $l = 0.5\eta\delta$ to within a certain constant factor η.

This constant factor η will no longer depend upon the value of δ and can be determined once and for all by experiment for a medium of the given type, e.g. for a cemented or an uncemented sandstone or limestone, etc. For this it is necessary to measure the actually obtaining local velocity, but such measurements are in practice extremely difficult.

The motion of a fluid only in a single pore and in the few neighbouring ones is connected by the Navier-Stokes equations, whilst the motion in a pore remote from that under consideration will not be subject to the same relation because the distribution of velocity, normal and tangential to an arbitrary plane section, is determined by the random microstructure of the porous medium, and one can postulate the hypothesis concerning the normal law of distribution of all the velocity characteristics of the flow. In this case the knowledge of the mean value of the velocity and its dispersion completely determines the values of the local velocities.

5. Equation (1.3) can be written in a fixed system of coordinates as

$$m\frac{\partial\psi}{\partial t}=\frac{\partial}{\partial x_1}\left(\lambda_1 w\frac{\partial\psi}{\partial x_1}\right)+\frac{\partial}{\partial x_2}\left(\lambda_2 w\frac{\partial\psi}{\partial x_2}\right)+\frac{\partial}{\partial x_3}\left(\lambda_3 w\frac{\partial\psi}{\partial x_3}\right)-w\frac{\partial\psi}{\partial x_1} \qquad (5.1)$$

where m is the porosity, w is the velocity of filtration ($w = m\bar{u}$), the axis of x_1 is directed along the vector w, and $\lambda_2 = \lambda_3$.

Equation (5.1) is valid for a homogeneous flow. In the general case it has the form:

$$\frac{\partial\psi}{\partial t}=\frac{\partial}{\partial x_i}\left(D_{ij}\frac{\partial\psi}{\partial x_j}\right)-\bar{u}_i\frac{\partial\psi}{\partial x_i} \qquad (5.2)$$

If in the porous medium there is a homogeneous filtration current and at some arbitrary point we add an indicator with a discharge Q of elements in unit time, then in a certain time the indicator becomes dispersed, and the lines of equal concentration form a set of "pear-shaped" curves. The shape of these curves is characteristic of the diffusion parameters of the porous medium.

For the solution of problems concerning the fixed-point source concentration in a homogeneous filtration flow, we can make use of analogous results in turbulent diffusion [7]. The average (nonrelative) concentration at the point x_1, x_2 or x_3 at the instant of time t is equal to the sum of the concentrations arising from a source after a time from t_0 to t, where t_0 is the time of start of operation of the source:

$$\Phi=\int_{t_0}^{t}Q\psi(x_1,x_2,x_3,t-\alpha)\,d\alpha= \qquad (5.3)$$

$$=\int_{t_0}^{t}\frac{Q\,(2\pi)^{-3/2}}{(\overline{x_1^2}\,\overline{x_2^2}\,\overline{x_3^2})^{1/2}}\exp\left\{-\frac{1}{2}\left[\frac{(x_1-\bar{u}\,(t-\alpha))^2}{\overline{x_1^2}}+\frac{x_2{}^2}{\overline{x_2^2}}-\frac{x_3{}^2}{\overline{x^2}_3}\right]\right\}d\alpha$$

In the case of a continuously acting source we have to set $t_0 = -\infty$. We thus obtain the steady distribution of relative concentration presented in the paper [7]. In the same way the solution can be obtained for the problem of the steady distribution of mean concentration arising from an infinite line diffusion source. This problem has practical importance in filtration* – namely, such conditions arise in the plane homogeneous flow of a fluid in a stratum of finite thickness, when the indicator is added through one of a series of bore-holes uniformly placed through the thickness of the stratum. Measuring the concentration at at least two other bore-holes, we can determine the field of concentration, and consequently also the diffusion parameters of the stratum. It is

* The author is greatly indebted to V.M. Shestakov for consideration of this question.

assumed that through all the said bore-holes no significant quantity of fluid either enters or leaves the stratum, in order not to disturb the conditions of homogeneity of the flow.

The solution has the form [7] :

$$\Phi = \frac{Qm}{2\pi w \sqrt{\lambda_1 \lambda_2}} \exp\left(-\frac{x_1}{2\lambda_1}\right) K_0\left(\frac{1}{2}\sqrt{\frac{x_1^2}{\lambda^2_1} + \frac{x_2^2}{\lambda_1 \lambda_2}}\right) \tag{5.4}$$

Let us seek such a solution of the homogeneous linear problem of mixing of a dye with the main fluid in motion in a porous medium. In this case equation (5.3) will have the following form:

$$m\frac{\partial \psi}{\partial t} = \lambda_1 w \frac{\partial^2 \psi}{\partial x^2} - w\frac{\partial \psi}{\partial x}, \qquad -\infty < x < \infty, \qquad t \geqslant 0 \tag{5.5}$$

with the initial condition

$$\psi(x, 0) = f(x) \tag{5.6}$$

The solution is easy to find if we reduce equation (5.5) to the general equation of heat conduction by the substitution

$$y = x - \frac{s}{m}, \qquad s = \int_0^t w(t)\, dt \tag{5.7}$$

and make use of the representation of the solution of the equation in the form of a Poisson integral. Then in the case of a rectangular shape of the initial distribution of the concentration, $f(y) = 0$ when $|y| > a$ and $f(y) = 1$ when $|y| \leqslant a$, the solution will have the following form:

$$\Phi = \int^t Q\psi(x_1, x_2, x_3, t - \alpha)\, d\alpha =$$

The formula (5.8) is found to be completely confirmed by the results of experiments [13] , where the coefficient of diffusion $D = \lambda_1 w$ was determined at five experimental points $\psi(0, t)$:

$$D_1 = 2.60 \cdot 10^{-2}, \qquad D_2 = 1.70 \cdot 10^{-2}, \qquad D_3 = 1.84 \cdot 10^{-2},$$
$$D_4 = 1.80 \cdot 10^{-2}, \qquad D_5 = 1.65 \cdot 10^{-2} \text{ см}^2 \text{ сек}^{-1}$$

The mean value of the diffusion coefficient $D_m = 1.92 \times 10^{-2}\,\text{cm}^2\text{sec}^{-1}$, and the mean value of λ_1 is equal to 0.107 cm.

Accordingly, $D_m \gg D_0 \approx 10^{-4} - 10^{-5}\,\text{cm}^2\text{sec}^{-1}$ which confirms the essentially different nature of the mixing from molecular diffusion.

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BIBLIOGRAPHY

1. Devison, B.B., Dvizhenie gruntovykh vod (The motion of ground waters). In the symposium: *Nekotorye novye voprosy mekhaniki sploshnoi sredy (Certain New Questions in the Mechanics of a Continuous medium)*. (Edited by Kochin, N.E.). Izd. Akad. Nauk SSSR, 1938.

2. Nikolaevskii, V.N., Kapilliarnaia model' diffuzii v poristykh sredakh (The capillary model of diffusion in porous media). Izv. Akad. Nauk SSSR, Otdel. Tekh. Nauk, *ser. mekh. i mash.* No. 4, 1959.

3. Scheidegger, A.E., Statistical hydrodynamics in porous media. *J. Appl. Phys.* Vol. 25, No. 8, August, 1954.

4. Batchelor, G., *Teoriia odnorodnoi turbulentnosti (Theory of homogeneous turbulence)*. Izd. inost. lit., 1955. Published in English.

5. Zhukovskii, N.E., Teoreticheskoe issledovanie o dvizhenii podpochvennykh vod (Theoretical investigation of the motion of subsoil waters) (1889). *Poln. sobr. soch.* (Complete Works). Vol. 7, Moscow, 1937.

6. Sedov, L.I., *Metody podobiia i razmernosti v mekhanike (Methods of similarity and dimensionality in mechanics)*. Gostekheoretizdat, 1958.

7. Fraenkel, F.N., Turbulentnaia diffuziia; raspredelenie srednei kontsentratsii v pole odnorodnogo turbulentnogo potoka (Turbulent diffusion; the distribution of mean concentration in the field of a homogeneous turbulent stream). Symposium "*Problemy mekhaniki*" (*Problems of Mechanics*). Izd. inostr. lit., 1955. Published in English.

8. Krasnopol'skii, A.A., Gruntovye i artezianskie kolodtsy (Soils and artesian wells). Gorn. zh. Nos. 3-7, March-July, 1912.

9. Aronofsky, J.S. and Heller, J.P., A diffusion model to explain mixing of flowing fluids in porous media. *J. Petrol. Tech.* December, 1957.

10. Baum, V.A., Issledovanie protsessa peremeshivaniia v potoke zhidkosti, protekaiushchei v trybakh, zapolnennykh kuskovym materialom (Investigation of the process of mixing in a stream of fluid flowing through pipes filled with granular material). *Izv. Akad. Nauk SSSR, Otdel. Tekh. Nauk*, No. 9, 1953.

11. Rosenberg, D.V. von, Mechanics of steady-state single-phase fluid displacement from porous media. *Amer. Inst. chem. Engng. J.* Vol. 2, No. 1, March, 1956.

12. Taylor, G., Dispersion of soluble matter in solvent flowing slowly through a tube. *Proc. roy. Soc.* A, Vol. 219, 1953.

13. Koch, H.A. and Slobod, R.L., Miscible slug process. *J. Petrol. Tech.*, February, 1957.

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